**Matrices types**

1. Zero matrix => [all items are zeros]
2. Identity matrix => [main diagonal are ones]
3. Column matrix (Vector) => [m \* 1]
4. Row matrix (Vector) => [1 \* n]
5. Square matrix => [m = n]
6. Diagonal matrix => [main diagonal contains integers, other are zeros]
7. Upper triangular matrix => [main diagonal and upper is integers, other are zeros]
8. Lower triangular matrix => [main diagonal and lower is integers, other are zeros]
9. Symmetric matrix => [lower triangular equal upper triangular]

**Addition & scalar multiplication properties**

1. A + B = B + A
2. A + (B + C) = (A + B) + C
3. (s1 \* s2) A = s1 (s2 \* A) = s2 (s1 \* A)
4. I \* A = A
5. s1 (A + B) = s1 \* A + s2 \* B
6. (s1 + s2) A = s1 \* A + s2 \* A
7. A + 0 = A
8. A + (-A) = 0
9. If s1 \* A = 0 => s1 = 0 or A = 0

\*\* s1 & s2 => scalar

**Multiplication properties**

1. A (B \* C) = (A \* B) C
2. A (B + C) = AB + AC
3. (A + B) C = AC + BC
4. s1 (A \* B) = (s1 \* A) B = A (s1 \* B)
5. AB ≠ BA
6. A \* I = I \* A = A
7. If AC = BC => A = B

\*\* s1 & s2 => scalar

\*\* I => identity matrix

**Power properties**

1. Ar \* As = Ar+s
2. (Ar)s = Ar\*s
3. A0 = 1

\*\* r & s => non-negative values

**Transpose properties**

1. (A+ B + C) T = AT + BT + CT
2. (s1 \* A) T = s1 \* AT
3. (A \* B \* C) T = CT \* BT \* AT
4. (AT)T = A

\*\* where s1 => scalar

\*\* T => transpose (convert columns to rows & convert rows to columns)

**Trace properties**

1. Matrix MUST be square.
2. tr (A) => [summation of main diagonal]
3. tr (A + B) = tr (A) + tr (B)
4. tr (AB) = tr (BA)
5. tr (s1 \* A) = s1 tr (A)
6. tr (AT) = tr (A)

\*\* s1 => scalar

**Adjoint VS cofactor matrices**

1. adj(A) = cof(A)T & cof(A) = adj(A)T
2. Suppose matrix A =

adj(A) = cof(A) =

Interchange items

Interchange items  
& change signs

Interchange items

change signs

1. Suppose matrix A =

cof(A) = adj(A) = cof(A)T

**Inverse properties**

1. Matrices MUST be square
2. If A \* B = B \* A = I => B is inverse of A or A is inverse of B
3. If A has no inverse => called singular
4. General rule: A-1 = \* adj(A) or \* cof(A)T
5. Inverse rule of A (2 x 2) = => A-1 = & det(A) = ad – bc
6. Inverse rule of A (3 x 3) =

Try to convert original matrix to identity matrix using row exchange, multiply row by factor, row athematic operation

If original matrix cannot convert to identity => its singular matrix & has no inverse.

1. (A-1)-1 = A
2. (Ak)-1 = (A-1) k = A-k
3. (s1 \* A)-1 = (A-1)
4. (AT)-1 = (A-1) T
5. (AB)-1 = B-1 \* A-1

**Linear system**

1. The greatest power in all equations (not solution) must be 1
2. Homogeneous system is equations with zeros in result.
3. Linear system has:
4. No solution => zero row has integer solution (lines parallel)
5. One solution => # of columns = solution elements (intersect between lines in one point)
6. Many solution => # of columns > solution elements OR there is a zero row with zero result

(lines are identical) AND we can suppose some variables with ( r, t, …. )

**Methods of solving linear systems (3 Ways)**

**1- Gaussian elimination**

1. Convert matrix to upper triangular matrix (called row echelon form) or closer.
2. In upper triangular => Back substitution from bottom to top  
   in lower triangular => Front substitution from top to bottom.
3. If there is a row all elements have been zeros, it must be the last row.
4. After back substitution, must substitute in original equations by result values.

**2- Gauss-Jordan elimination**

1. Convert matrix to upper & lower triangular matrix (called reduced row echelon form) or closer.
2. In upper triangular => Back substitution from bottom to top  
   in lower triangular => Front substitution from top to bottom.
3. If there is a row all elements have been zeros, it must be the last row.
4. After back substitution, must substitute in original equations by result values.

**3- Crammer role**

Suppose = => where Ax = b

1. Calculate det (A)
2. Change column 1 with solution & calculate det (A1)
3. Change column 2 with solution & calculate det (A2)
4. Change column 3 with solution & calculate det (A3)
5. Calculate x1 = & x2 = & x3 =

**Determinants**

1. Calculated on square matrices only.
2. If |A| ≠ 0 => A is invertible A-1 is existing A is non-singular matrix
3. If det (A) = 0 => A is non-invertible A-1 is not existing A is singular matrix
4. det (AB) = det (A) \* det (B)
5. |C A| = Cn \* |A| => A is matrix & C is constant & n is multiple coefficient
6. |AT| = |A|
7. |A-1| =
8. If matrix B obtain from exchange two rows or columns in matrix A => det (A) = -det (B)
9. If matrix B obtain from row operations in matrix A => det (A) = det (B)
10. If matrix B obtain from multiply rows in matrix A by non-zero constant => det (B) = C det (A)
11. If there is row or column consist of zeros => det (A) = 0
12. If there are rows or columns are equal => det (A) = 0
13. If there is one row or column is multiple of another => det (A) = 0

**Determinants Calculation methods**

1. det (A) = ad – bc => A =
2. det (A) = => (a \* e \* i) + (b \* f \* g) + (c \* d \* h) – (b \* d \* i) – (a \* f \* h) – (c \* e \* g)
3. Using general method (minor & cofactor)

A = => det (A) = +a \* – b \* + c \*

Minor M11 = +a \* , Cofactor C11 = (-1)1+1 \* M11

Notes:

1. Signs of first row are (+ - +) & second row MUST start with (–) regardless sign of last item in previous row.
2. If didn’t dedicate which row must use, it preferring use row that have maximum items of zeros.
3. If matrix upper or lower triangular, det = multiplication of main diagonal.
4. Coefficient take from each row only, if take more than one coefficient then total of them calculate by multiplication.